

Substitutability of Capital, Investment Costs and Foreign Aid[♦]

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This paper was written to honor the contributions of John Pitchford, an innovative scholar, who among other things, produced the first published analysis of the CES production function. Stephen Turnovsky looks back with pleasure to the 1970s when John and he were colleagues at the ANU and enjoyed many fruitful collaborations. The authors would like to thank three anonymous readers for their comments. This research was supported in part by the Castor Endowment at the University of Washington. In addition, Chatterjee gratefully acknowledges financial support from the Grover and Creta Ensley Fellowship.

Abstract

This paper explores the relationship between foreign aid and public investment in the context of intertemporal growth. The static models in the existing development literature have predominantly neglected the intertemporal behavior of savings and investment in response to aid flows, and their consequent impact on growth and transitional dynamics. On the other hand, the macroeconomics literature has generally neglected the issue of external financing of public investment from official sources such as foreign aid and capital transfers. Moreover, an over-riding assumption in both strands of literature is that of a Cobb-Douglas production function that has the characteristic that the intratemporal elasticity of substitution between inputs is unity. Recent empirical evidence, however, points toward the CES production function as a better approximation for production structures in growing economies. This paper examines the dynamic consequences of financing public investment by foreign aid or capital transfers in an intertemporal optimizing growth framework by employing a CES production structure. We conduct a numerical analysis of the transitional dynamics of such an economy and characterize the trade-off between the degree of substitutability between public and private capital, cost of investment, and intertemporal welfare in response to both tied and untied aid programs.

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1. Introduction

Public investment is widely accepted as being a crucial determinant of economic growth. Interest in the impact of public capital on private capital accumulation and economic growth originated with the seminal theoretical work of Arrow and Kurz (1970) and the more recent empirical research of Aschauer (1989a, 1989b).¹ Most of the subsequent literature has focused on closed economies, using both the Ramsey model and the AK endogenous growth framework; see e.g. Futagami, Morita, and Shibata (1993), Glomm and Ravikumar (1994), Baxter and King (1993), Fisher and Turnovsky (1998). Turnovsky (1997a) extends Futagami et al. to a small open economy and introduces various forms of distortionary taxation, as well as the possibility of both external and internal debt financing. Devarajan, Xie, and Zou (1998) address the issue of whether public capital should be provided through taxation or through granting subsidies to private providers.²

A critical issue, especially in poor, resource-constrained developing countries, concerns how the new investment in infrastructure is financed. One significant source for funding such investment is external financing. This may be in the form of borrowing from abroad, through bilateral or multilateral loans, or through unilateral capital transfers, in the form of tied grants or official development assistance, as recently observed in the European Union. Faced with below average per-capita incomes and low growth rates among some of its joining members, the EU introduced pre-accession aid programs to assist these and other potential member nations in their transition into the union³. This process of “catching up” began in 1989 with a program of unilateral capital transfers from the EU through the Structural Funds program, and subsequent programs were introduced in 1993 and in 2000. These assistance programs tied the capital transfers (or grants) to the accumulation of public capital, and were aimed at building up infrastructure in the recipient nation. The objective of these aid programs was for the recipient economy to attain strong positive growth

¹See Gramlich (1994) for a comprehensive survey of the recent empirical literature.

²The efficient use of infrastructure is a further important issue. For example, Hulten (1996) shows that inefficient use of infrastructure accounts for more than 40 percent of the growth differential between high and low growth countries.

³Greece, Ireland, Spain, and Portugal were recipients of unilateral capital transfers tied to public investment projects under the Structural Funds Program between 1989-1993 and 1993-1999. A similar tied transfer program, called Agenda 2000, has been initiated for eleven aspiring member nations (Central Eastern European Countries), and is expected to continue until 2006; see European Union (1998a, 1998b).

differentials relative to the EU average in the short run, and thereby achieve higher and sustainable living standards in alignment with EU standards, and ultimately to gain accession to EU membership.

In a recent paper, Chatterjee, Sakoulis, and Turnovsky (2001) have analyzed the process of developmental assistance in the form of tied-capital transfers to a small growing open economy. One critical assumption adopted in that analysis is that the underlying production function is of the Cobb-Douglas form in private and public capital. While this functional form is prevalent throughout much of the recent endogenous growth literature, it is of course restrictive; see Lucas (1988), Barro (1990), Futagami et al. (1993), Bond, Wang, and Yip (1996), and Turnovsky (1997a). In particular, it suffers from the serious shortcoming that the resulting impact of the transfer on the growth performance is predicated on the intratemporal elasticity of substitution between these two forms of capital being assumed to be unity. Intuitively, one would expect the impact of a tied transfer to be highly sensitive to the degree of intratemporal substitution between these two types of capital inputs. To analyze this, one needs to employ a more flexible production specification, such as the constant elasticity of substitution (CES) production function, which accommodates alternative degrees of substitution. This is the task undertaken in the present paper. Indeed, as our analysis will confirm, the elasticity of substitution is an important determinant of both the dynamic adjustment paths generated by a program of tied-transfers and their welfare implications.

The CES production function has a long history, being initially introduced by Pitchford (1960), and Arrow, Chenery, Minhas, and Solow (1961). The original specification was in terms of capital and raw labor, and extensive empirical evidence on the elasticity of substitution between these two inputs was produced during the 1960's and 1970's. Berndt (1976) provides a reconciliation between alternative estimates for the aggregate production function, concluding that estimates generally range between around 0.8 and 1.2. In a recent panel study of 82 countries over a 28-year period, Duffy and Papageorgiou (2000) find that they can reject the Cobb-Douglas specification for the entire sample in favor of the more general CES production function. They also report that the degree of substitution between inputs (in their case human and physical capital) may vary with the stages of development. For example, there is a higher degree of substitutability of

inputs in rich countries than in poor countries, a feature absent from the Cobb-Douglas specification. Empirical evidence on the substitutability of public and private capital is sparse. Lynde and Richmond (1993) introduce public and private capital into a more general translog production function for U.K. manufacturing and find that the Cobb-Douglas specification is rejected.

Factor substitution can occur intratemporally and/or intertemporally. Whereas the former is incorporated by the CES production function, the latter may be captured by the introduction of differential costs of adjustment, along the lines associated with Hayashi (1982). Indeed, the impact of foreign aid on the evolution of the economy depends not only on the short-run degree of substitutability between the two types of capital, but also on their relative costs of adjustment.

This paper attempts to bridge the gap between the development literature on the impact of foreign aid and the growth literature on the role of public investment, in the context of a growing open economy that receives development assistance in the form of foreign aid from the rest of the world. Specifically, our paper contributes to the above branches of literature in two important directions. First, we consider aid in the form of tied unilateral capital transfers, i.e., funds to be used by the recipient for the specific purpose of creating public capital.⁴ As Brakman and van Marrewijk (1998) point out, in the post World War II era, unilateral capital transfers have increasingly taken the form of development assistance or foreign aid. This is important when one recognizes that between two-thirds and three-fourths of official development assistance to infrastructure is fully or partially tied.⁵ On the other hand, most of the existing development literature, which examines the possible effects of aid on saving and investment in developing countries, has been based mainly on static models⁶. In contrast, we embed the aid flow in an intertemporal optimization framework characterized by endogenous growth, which enables us to compare both the short-run and the long-

⁴ Bhagwati (1967) points out that tied assistance may take different forms. The transfer or aid from abroad may be linked to a (i) specific investment project, (ii) specific commodity or service, or (iii) to procurement in a specific country. We focus our analysis on the first type of tying, i.e. to an investment project. Examples of such tied capital transfers include the relocation of German capital equipment at the end of the Second World War to Eastern Europe and the Soviet Union, the Marshall Plan in the post-World War II era for the reconstruction of Europe, and more recently, the European Union's pre-accession aid programs for aspiring member nations.

⁵ World Bank (1994).

⁶ See Cassen (1986), and more recently, Brakman and van Marrewijk (1998) for a survey of this literature. Two exceptions include Djajic, Lahiri, and Raimondos-Moller (1999), and Hatzipanayotou and Michael (2000), who examine the effects of transfers in an intertemporal context.

run effects of tied and untied aid on the dynamic evolution and growth rate of the economy, and ultimately on welfare⁷.

Second, since it is likely that external assistance and borrowing will fail to meet the total financial needs for public investment, domestic participation by both the government and the private sector is also important. Recently, in a panel study of 56 developing countries and six four-year periods (1970-93), Burnside and Dollar (2000) find that foreign aid is most effective when combined with a positive policy environment in the recipient economy. In earlier works, Gang and Khan (1991) and Khan and Hoshino (1992) report that most bilateral aid for public investment in LDCs is tied and is given on the condition that the recipient government invests certain resources into the same project. We specifically characterize the consequences of domestic co-financing of public investment and outline the trade-offs faced by a recipient government when it responds optimally to a flow of external assistance from abroad.

In addition to the CES specification of technology, the model we employ has the following key characteristics. First, external assistance is tied to the accumulation of public capital, which is therefore an important stimulus for private capital accumulation and growth. Second, new investment in both types of capital is subject to convex costs of installation. Allowing for differential costs of investment for public and private capital raises the issue of how the degree of substitutability between the two capital stocks interact with installation costs in determining the effect of a tied foreign aid shock. Third, we assume that public investment in infrastructure is financed both by the domestic government, as well as via the flow of international transfers, thereby incorporating the important element of domestic co-financing, characteristic of most bilateral aid programs that are tied to specific public investment projects. The international transfers are assumed to be tied to the scale of the recipient economy and therefore are consistent with maintaining an equilibrium of sustained (endogenous) growth in that economy.

⁷ This issue is also related to the pure “transfer problem”, one of the classic issues in international trade, and dates back to Keynes (1929) and Ohlin (1929). Recent contributions include Bhagwati et. al. (1983), Galor and Polemarchakis (1987), Turunen-Red and Woodland (1996), and Djajic et. al. (1999). For a comprehensive survey of the literature, see Brakman and Marrewijk (1998). Our analysis differs from this literature by focusing on “productive” (tied) transfers, the use of which is tied to public investment.

We also assume that the small open economy faces restricted access to the world capital market in the form of an upward-sloping supply curve of debt, according to which the country's cost of borrowing depends upon its debt position, relative to its total capital stock, the latter serving as a measure of its debt-servicing capability. This assumption is motivated by the large debt burdens of most developing countries, which give rise to the potential risk of default on international borrowing. Indeed, evidence suggesting that more indebted economies pay a premium on their loans from international capital markets to insure against default risk has been provided by Edwards (1984). An interesting question, therefore, is whether barriers to international borrowing have any implications for the welfare effects of foreign aid programs.

The main results of our model are the following. The effect of an increase in foreign aid depends critically on whether it is tied or untied. An untied aid program does not generate any dynamic response, but instead leads to instantaneous increases in consumption and welfare. On the other hand, an aid program that is tied to investment in public capital generates a transitional dynamic adjustment in the recipient economy. The magnitude and the direction of the transitional dynamics and long run effects depend crucially upon the elasticity of substitution between the two types of capital in the recipient economy. Our analysis suggests that tied aid is more effective in terms of its impact on long-run growth and welfare for countries that have low substitutability between factors of production. This finding has important policy implications, especially in light of recent empirical evidence suggesting that less developed or poor countries have elasticities of substitution that are significantly below unity. We find that the welfare gains from a particular type of aid program (tied or untied) are sensitive to the costs of installing public capital and capital market imperfections, even for small changes in the degree of substitutability between inputs. Economies in which the elasticity of substitution between the two types of capital and the installation costs are relatively high, are likely to find tied transfers to be welfare-deteriorating. For such economies untied aid will be more appropriate.

The rest of the paper is organized as follows. The analytics of the theoretical model are laid out in Section 2. Section 3 presents a numerical analysis of the impact of a foreign aid shock and the resulting transitional dynamics. Section 4 briefly addresses the issue of co-financing, while Section

5 discusses the sensitivity of intertemporal welfare to the elasticity of substitution, investment costs, and capital market imperfections. Section 6 presents some concluding remarks.

2. The Analytical Framework

2.1 Private Sector

We consider a small open economy populated by an infinitely-lived representative agent who produces and consumes a single traded commodity. Output, Y , of the commodity is produced using the Constant Elasticity of Substitution (CES) production function

$$Y = \alpha \left[\eta K_G^{-\rho} + (1-\eta) K^{-\rho} \right]^{-1/\rho}; \quad \alpha > 0, \quad 0 < \eta < 1, \quad \rho > -1 \quad (1a)$$

where K denotes the representative agent's stock of private capital, K_G denotes the stock of public capital, and $\sigma \equiv 1/(1+\rho)$ is the elasticity of substitution between private and public capital in production. The model abstracts from labor so that private capital should be interpreted broadly to include human, as well as physical capital; see Rebelo (1991).

The agent consumes this good at the rate C , yielding utility over an infinite horizon represented by the isoelastic utility function:⁸

$$U \equiv \int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\beta t} dt; \quad -\infty < \gamma < 1 \quad (1b)$$

The agent also accumulates physical capital, with expenditure on a given change in the capital stock, I , involving adjustment (installation) costs specified by the quadratic (convex) function

$$\psi(I, K) = I + h_1 \frac{I^2}{2K} = I \left(1 + h_1 \frac{I}{2K} \right) \quad (1c)$$

This equation is an application of the familiar cost of adjustment framework, where we assume that the adjustment costs are proportional to the *rate* of investment per unit of installed capital (rather

⁸ The exponent γ is related to the intertemporal elasticity of substitution s , by $s = 1/(1-\gamma)$, with $\gamma = 0$ being equivalent to a logarithmic utility function.

than its level). The linear homogeneity of this function is necessary for a steady-state equilibrium having ongoing growth to be sustained. The net rate of capital accumulation is thus:

$$\dot{K} = I - \delta_K K \quad (1d)$$

where δ_K denotes the rate of depreciation of private capital.

Agents may borrow internationally on a world capital market. The key factor we wish to take into account is that the creditworthiness of the economy influences its cost of borrowing from abroad. Essentially we assume that world capital markets assess an economy's ability to service debt costs and the associated default risk, the key indicator of which is the country's debt-capital (equity) ratio. As a result, the interest rate countries are charged on world capital markets increases with this ratio. This leads to the upward sloping supply schedule for debt, expressed by assuming that the borrowing rate, $r(N/K)$, charged on (national) foreign debt, N , relative to the stock of private capital, K , is of the form:

$$r(N/K) = r^* + \omega(N/K); \quad \omega' > 0 \quad (1e)$$

where r^* is the exogenously given world interest rate, and $\omega(N/K)$ is the country-specific borrowing premium that increases with the nation's debt-capital ratio. The homogeneity of the relationship is required to sustain a balanced growth equilibrium.⁹

The agent's decision problem is to choose consumption, and the rates of accumulation of capital and debt, to maximize intertemporal utility (1b) subject to the flow budget constraint

$$\dot{N} = C + r(N/K)N + \Psi(I, K) - (1 - \tau)Y + \bar{T} \quad (2)$$

⁹ A rigorous derivation of (1e) presumes the existence of risk. Since we do not wish to model a full stochastic economy, we should view (1e) as representing a convenient reduced form, one supported by empirical evidence; see e.g. Edwards (1984) who finds a significant positive relationship between the spread over LIBOR (e.g. r^*) and the debt-GNP ratio. Eaton and Gersovitz (1989) provide formal justifications for the relationship (1e). Various formulations can be found in the literature. The original formulation by Bardhan (1967) expressed the borrowing premium in terms of the absolute stock of debt; see also Obstfeld (1982), Bhandari, Haque, and Turnovsky (1990). Other authors such as Sachs (1984) also argue for a homogeneous function such as (1e). We have also considered the Edwards (1984) formulation, $r = r(N/Y)$, and very similar results to those reported are obtained.

where N is the stock of debt held by the private sector, τ is the income tax rate, and \bar{T} denotes lump-sum taxes.¹⁰ It is important to emphasize that in performing his optimization, the representative agent takes the borrowing rate, $r(\cdot)$ as given. This is because the interest rate facing the debtor nation, as reflected in its upward sloping supply curve of debt, is a function of the economy's *aggregate* debt-capital ratio, which the individual agent assumes he is unable to influence.

The optimality conditions with respect to C and I are respectively

$$C^{\gamma-1} = \nu \quad (3a)$$

$$1 + h_1(I/K) = q \quad (3b)$$

where ν is the shadow value of wealth in the form of internationally traded bonds, q' is the shadow value of the agent's private capital stock, and $q = q'/\nu$ is defined as the market price of private capital in terms of the (unitary) price of foreign bonds. The first of these conditions equates the marginal utility of consumption to the shadow value of wealth, while the latter equates the marginal cost of an additional unit of investment, which is inclusive of the marginal installation cost $h_1 I/K$, to the market value of capital. Equation (3b) may be immediately solved to yield the following expression for the rate of private capital accumulation

$$\frac{\dot{K}}{K} \equiv \phi_K = \frac{q-1}{h_1} - \delta_K \quad (3b')$$

Applying the standard optimality conditions with respect to N and K implies the usual arbitrage relationships, equating the rates of return on consumption and investment in private capital to the costs of borrowing abroad

$$\beta - \frac{\dot{\nu}}{\nu} = r\left(\frac{N}{K}\right) \quad (4a)$$

$$\frac{(1-\tau)(1-\eta)\alpha[\eta(K_G/K)^{-\rho} + (1-\eta)]^{(1+\rho)/\rho}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2h_1 q} - \delta_K = r\left(\frac{N}{K}\right) \quad (4b)$$

¹⁰ It is natural for us to assume $N > 0$, so that the country is a debtor nation. However, it is possible for $N < 0$ in which case the agent accumulates credit by lending abroad. For simplicity, interest income is assumed to be untaxed.

Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must hold:

$$\lim_{t \rightarrow \infty} vB e^{-\beta t} = 0; \quad \lim_{t \rightarrow \infty} q' K e^{-\beta t} = 0. \quad (4c)$$

2.2 Public Capital, Transfers, and National Debt

The resources for the accumulation of public capital come from two sources: domestically financed government expenditure on public capital, \bar{G} , and a program of capital transfers, TR , from the rest of the world. We therefore postulate

$$G \equiv \bar{G} + \lambda TR \quad 0 \leq \lambda \leq 1$$

where λ represents the degree to which the transfers from abroad are tied to investment in the stock of public infrastructure. The case $\lambda=1$ implies that transfers are completely tied to investment in public capital, representing a “productive” transfer. In the other polar case, $\lambda=0$, incoming transfers are not invested in public capital and hence represent a “pure” transfer, of the Keynes-Ohlin type.

We assume that the gross accumulation of public capital, G , is also subject to convex costs of adjustment, similar to that of private capital¹¹

$$\Omega(G, K_G) = G \left(1 + (h_2/2) (G/K_G) \right).$$

In addition, the stock of public capital depreciates at the rate δ_G so that the net rate of public capital accumulation is,

$$\dot{K}_G = G - \delta_G K_G. \quad (5)$$

To sustain an equilibrium of on-going growth, both domestic government expenditure on infrastructure (\bar{G}) and the flow of transfers from abroad must be tied to the scale of the economy

$$\bar{G} = \bar{g}Y, \text{ and } TR = \theta Y, \quad 0 < \bar{g} < 1, \quad \theta > 0, \quad 0 < \bar{g} + \theta < 1$$

¹¹Noting the definition of G , we see that the transfers contribute to the financing of the installation costs, as well as to the accumulation of the new public capital.

We can therefore rewrite (5) in the following form

$$\dot{K}_G = G - \delta_G K_G = gY - \delta_G K_G = (\bar{g} + \lambda\theta)Y - \delta_G K_G; g = \bar{g} + \lambda\theta > 0 \quad (5')$$

and dividing (5) by K_G , the growth rate of public capital is given by

$$\frac{\dot{K}_G}{K_G} \equiv \phi_G = (\bar{g} + \lambda\theta) \frac{Y}{K_G} - \delta_G. \quad (6)$$

The government sets its tax and expenditure parameters to continuously maintain a balanced budget:

$$\tau Y + TR + \bar{T} = \Omega(G, K_G) \quad (7)$$

The national budget constraint, or the nation's current account, can be obtained by combining (7) and (2),

$$\dot{N} = r(N/K)N + C + \Psi(I, K) + \Omega(G, K_G) - Y - TR. \quad (8)$$

Equation (8) states that the economy accumulates debt to finance its total expenditures on public capital, private capital, consumption and interest payments net of output produced and transfers received. It is immediately apparent that higher consumption or investment raises the rate at which the economy accumulates debt. The direct effect of a larger unit transfer on the growth rate of debt is given by $(\lambda - 1) + (h_2/K_G)\lambda G$. An interesting observation is that the more transfers are tied to public investment (the higher λ), the lower the decrease in the growth rate of debt. When transfers are completely tied to investment in infrastructure, i.e., $\lambda=1$, debt increases due to higher installation costs. However, the indirect effects, induced by the change will still need to be taken into account.

2.3 Macroeconomic Equilibrium

The steady-state equilibrium has the characteristic that all real quantities grow at the same constant rate and that q , the relative price of capital, is constant. Thus we shall express the dynamics of the system in terms of the following stationary variables, normalized by the stock of private capital, $c \equiv C/K$, $k_g \equiv K_G/K$, $n \equiv N/K$, and q . The equilibrium system is derived as follows.

First, taking the time derivative of k_g and substituting (6) and (3b') yields

$$\frac{\dot{k}_g}{k_g} \equiv \phi_g - \phi_K = \alpha(\bar{g} + \lambda\theta) \left[\eta + (1-\eta)k_g^\rho \right]^{-1/\rho} - \frac{q-1}{h_1} - (\delta_G - \delta_K) \quad (9a)$$

Next, dividing (8) by N , and substituting, we can rewrite (8) as

$$\phi_N = r(n) + \frac{1}{n} \left[\{(\bar{g} + \lambda\theta) - (1+\theta)\} y + \frac{q^2-1}{2h_1} + \frac{h_2}{2} (\bar{g} + \lambda\theta)^2 \frac{y^2}{k_g} + c \right] \quad (8')$$

where $y = Y/K = \alpha \left[\eta k_g^{-\rho} + (1-\eta) \right]^{-1/\rho}$. Taking the time derivative of n and combining with (3b')

leads to:

$$\begin{aligned} \frac{\dot{n}}{n} \equiv \phi_N - \phi_K = r(n) + \frac{1}{n} \left[\{(\bar{g} + \lambda\theta) - (1+\theta)\} y + \frac{q^2-1}{2h_1} + \frac{h_2}{2} (\bar{g} + \lambda\theta)^2 \frac{y^2}{k_g} + c \right] \\ - \left(\frac{q-1}{h_1} \right) + \delta_K \end{aligned} \quad (9b)$$

Third, from (3a) and (4a), we derive the growth rate of consumption

$$\frac{\dot{C}}{C} = \phi_C = \frac{r(n) - \beta}{1 - \gamma}$$

Taking the time derivative of c and combining with the above expression leads to:

$$\frac{\dot{c}}{c} \equiv \phi_C - \phi_K = \frac{r(n) - \beta}{1 - \gamma} - \frac{q-1}{h_1} + \delta_K \quad (9c)$$

Finally, rewriting (4b) implies

$$\dot{q} = \left[r(n) + \delta_K \right] q - \alpha(1-\tau)(1-\eta) \left[\eta k_g^{-\rho} + (1-\eta) \right]^{-(1+\rho)/\rho} - \frac{(q-1)^2}{2h_1} \quad (9d)$$

Equations (9a) – (9d) provide an autonomous set of dynamic equations in k_g, n, c , and q , from which the steady-state equilibrium can be derived.

2.4 Steady State Equilibrium

The economy reaches steady state when $\dot{k}_g = \dot{n} = \dot{c} = \dot{q} = 0$, implying that $\dot{K}/K = \dot{K}_G/K_G = \dot{N}/N = \dot{C}/C \equiv \tilde{\phi}$, the steady-state growth rate of the economy. The steady state is thus described by:

$$\alpha(\bar{g} + \lambda\theta) \left[\eta + (1-\eta)\tilde{k}_g^\rho \right]^{-1/\rho} - \delta_G = \frac{\tilde{q}-1}{h_1} - \delta_K \quad (10a)$$

$$r(\tilde{n}) + \frac{1}{\tilde{n}} \left[\{(\bar{g} + \lambda\theta) - (1+\theta)\} \tilde{y} + \frac{\tilde{q}^2 - 1}{2h_1} + \frac{h_2}{2} (\bar{g} + \lambda\theta)^2 \frac{\tilde{y}^2}{\tilde{k}_g} + \tilde{c} \right] = \left(\frac{\tilde{q}-1}{h_1} \right) - \delta_K \quad (10b)$$

$$\left[r(\tilde{n}) + \delta_K \right] \tilde{q} - \alpha(1-\tau)(1-\eta) \left[\eta\tilde{k}_g^{-\rho} + (1-\eta) \right]^{-(1+\rho)/\rho} - \frac{(\tilde{q}-1)^2}{2h_1} = 0 \quad (10c)$$

$$\frac{r(\tilde{n}) - \beta}{1-\gamma} = \frac{\tilde{q}-1}{h_1} - \delta_K = \tilde{\phi} \quad (10d)$$

Equations (10a)-(10d) determine the steady-state equilibrium in the following recursive manner. First, equations (10a), (10c) and (10d) jointly determine $\tilde{k}_g, \tilde{q}, \tilde{r}(\cdot)$, and $\tilde{\phi}$, such that the equilibrium growth rates of public capital, private capital, and consumption are all equal, and that the rate of return on private capital equal the borrowing costs. Having determined \tilde{r} and \tilde{k}_g , the equilibrium stock of debt-capital ratio, \tilde{n} , is obtained from (1e). Given $\tilde{k}_g, \tilde{q}, \tilde{r}(\cdot)$, and \tilde{n} (and recalling the definition of y), the equilibrium consumption-capital ratio, \tilde{c} , is obtained from the current account equilibrium condition (10b). Provided $\tilde{r} > \tilde{\phi}$ (which we shall show below is required for the transversality condition to hold) higher marginal borrowing costs reduce total interest payments raising the consumption-capital ratio. Also, higher installation costs, h_2 , reduce the amount of output available for consumption, \tilde{c} . Because this system is highly non-linear, it need not be consistent with a well-defined steady state equilibrium with $\tilde{k}_g > 0, \tilde{c} > 0$. Our numerical simulations, however, yield well-defined steady state values for all plausible specifications of all the structural and policy parameters of the model.¹²

¹²A discussion of issues pertaining to non-existent or multiple equilibria in a related model is provided by Turnovsky (2000). Similar issues apply here.

It is seen that the transfers impinge on the equilibrium through the growth of public capital (10a) and the goods market equilibrium (10b). Setting $\lambda = 0$ we see from (10a), (10c) and (10d) that $\tilde{k}_g, \tilde{q}, \tilde{r}(\cdot)$, and $\tilde{\phi}$ are all independent of the level of untied transfers θ , an increase in which is fully reflected in steady-state consumption. If the transfers are tied, they will lead to an increase in the steady-state ratio of public to private capital, growth rate, and debt-capital ratio, by an amount that depends upon the elasticity of substitution. In the extreme case of perfect substitutability between the two types of capital ($\rho = -1$) $\tilde{q}, \tilde{r}(\cdot)$, and $\tilde{\phi}$ are all independent of θ , while k_g increases.

2.5 Equilibrium Dynamics

Equations (9a) - (9d) form the dynamics of the system in terms of k, n, q , and c . Linearizing these equations around the steady-state values of k_g, n, q , and c obtained from (10a) - (10d),

$$\begin{pmatrix} \dot{k}_g \\ \dot{n} \\ \dot{c} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 & -\tilde{k}_g/h_1 \\ a_{21} & r'(\tilde{n})\tilde{n} + r(\tilde{n}) - \tilde{\phi} & 1 & \frac{\tilde{q} - \tilde{n}}{h_1} \\ 0 & \frac{r'(\tilde{n})\tilde{c}}{1-\gamma} & 0 & -\tilde{c}/h_1 \\ a_{41} & r'(\tilde{n})\tilde{q} & 0 & r(\tilde{n}) - \tilde{\phi} \end{pmatrix} \begin{pmatrix} k_g - \tilde{k} \\ n - \tilde{n} \\ c - \tilde{c} \\ q - \tilde{q} \end{pmatrix} \quad (11)$$

where

$$a_{11} = \alpha^{-\rho} (\eta - 1) (\bar{g} + \lambda \theta) \frac{\tilde{y}^{1+\rho}}{\tilde{k}_g},$$

$$a_{21} = \alpha^{-\rho} \eta [(\bar{g} + \lambda \theta) - (1 + \theta)] \left(\tilde{y}/\tilde{k}_g \right)^{(1+\rho)} + h_2 \alpha^{-\rho} \eta (\bar{g} + \lambda \theta)^2 \left(\tilde{y}/\tilde{k}_g \right)^{(2+\rho)} - \frac{h_2}{2} (\bar{g} + \lambda \theta)^2 \left(\tilde{y}/\tilde{k}_g \right)^2,$$

$$\text{and } a_{41} = -\alpha^{-2\rho} \eta (1 - \tau) (1 - \eta) (1 + \rho) \left(\tilde{y}^{1+2\rho} / \tilde{k}_g^{1+\rho} \right).$$

The determinant of the coefficient matrix of (11) can be shown to be positive under the condition that $r(\cdot) > \tilde{\phi}$ i.e., the steady-state interest rate facing the small open economy must be greater than the steady-state growth rate of the economy. Imposing the transversality condition (4c), we see that this condition is indeed satisfied. Since (11) is a fourth-order system, a positive determinant implies that there could be 0, 2, or 4 positive (unstable) roots. However, our numerical simulations yield

saddle-point behavior for all plausible ranges of parameters. Thus the dynamic system (11) is saddle-point stable with two positive (unstable) and two negative (stable) roots, the latter being denoted by μ_1 and μ_2 , with $\mu_2 < \mu_1 < 0$.

3. Numerical Analysis of Transitional Dynamics

Due to the complexity of the model, we will employ numerical methods to examine the dynamic effects of transfers. We begin by calibrating a benchmark economy, using the following parameters representative of a small open economy, which starts out from an equilibrium with zero transfers.

The Benchmark Economy

Preference parameters:	$\gamma = -1.5, \beta = 0.04$
Production parameters:	$\alpha = 0.4, \eta = 0.2, h_I = 15, h_2 = 15$
Elasticity of substitution in production:	$\sigma = 0.33, 1, \rightarrow \infty$
Depreciation rates:	$\delta_K = 0.05, \delta_G = 0.04$
World interest rate:	$\bar{r} = 0.06,$
Premium on borrowing:	$a = 0.1^{13}$
Policy parameters:	$\tau = 0.15, \bar{g} = 0.05$
Transfers:	$\theta = 0, \lambda = 0$

Our choices of preference parameters β, γ , and depreciation rate, δ_K, δ_G , the world interest rate, \bar{r} are standard, while α is a scale variable. The productive elasticity of public capital $\eta = 0.2$ is consistent with the empirical evidence (see Gramlich, 1994). The borrowing premium $a = 0.10$ is chosen to ensure a plausible equilibrium national debt to income ratio. The tax rate is set at $\tau = 0.15$, while the rate of government expenditure on public investment is assumed to be $\bar{g} = 0.05$. The choice of adjustment costs is less obvious. Setting $h_I = 15$ is consistent with Origueira and Santos (1997), who find that $h_I = 16$ leads to a plausible speed of convergence of around 2%. Auerbach and Kotlikoff (1987) assume $h_I = 10$, recognizing that this is at the low values of estimates, while Barro and Sala-i-Martin (1995) propose a value above 10. We have also assumed

¹³ The functional specification of the upward sloping supply curve that we use is: $r(n) = \bar{r} + e^{an} - 1$. Thus, in the case of a perfect world capital market, when $a = 0$, $r = \bar{r}$, the world interest rate.

smaller values of h , with little change in results. Note also that the equality of adjustment costs between the two types of capital serves as a plausible benchmark.

The critical parameter upon which we focus is the elasticity of substitution, σ , and we consider three benchmark economies, depending on the degree of substitutability between public and private capital in production. These include: (i) low elasticity of substitution, $\sigma = 0.33$ (Table 1A), (ii) unitary elasticity of substitution, $\sigma = 1$ (Table 1B), and (iii) perfect substitutability between the two types of capital, where $\sigma \rightarrow \infty$ (Table 1C). Benchmark (ii) represents the familiar Cobb-Douglas production function, while (i) and (iii) represent two extreme cases, where there is very little or extremely high degree of substitutability in production.

The calibrated benchmark economy derived from the above parameter specification is reported in Table 1. The standard case of the Cobb-Douglas specification is reported in Table 1B, Row 1. It implies a steady-state ratio of public to private capital of 0.29; the consumption-output ratio is 0.60, the debt to GDP ratio of 0.45, leading to an equilibrium borrowing premium of 1.42% over the world rate. The capital-output ratio is over 3, with the equilibrium growth rate being around 1.37%. This equilibrium is a reasonable characterization of a small medium-indebted economy, experiencing a modest steady rate of growth and having a relatively small stock of public capital.

Parts 1A-1C reveal the sensitivity of the steady-state equilibrium to variations in the elasticity of substitution in production. For a very low degree of substitution in production, $\sigma = 0.33$ (Benchmark I, Table 1A, Row 1), the steady-state ratio of public to private capital is increased to 0.437, the interest rate is 2.4%, *lower* than the world interest rate of 6%, which implies that this economy is a net creditor to the rest of the world, and thus has an initial current account surplus. This is reflected in a debt-output ratio of -1.24 . The low elasticity of substitution causes agents to lower their investment in the stock of private capital, and enjoy higher consumption, leading to a consumption-output ratio of 0.78. Due to the low investment in private capital and high consumption, the steady-state growth rate in this economy is -0.6% .

In the extreme case of perfect substitutability between public and private capital (Benchmark III, Table 1C, Row 1), the equilibrium ratio of public to private capital decreases to 0.27. The consumption-output ratio decreases to 0.51 and the current account deficit increases, reflected in a

higher debt-GDP ratio of 1.11 and a steady-state interest rate of 9.87%. The high elasticity of substitution leads to an equilibrium growth rate of 2.35%.

3.1 *A Permanent Foreign Aid Shock: Long Run Effects*

We now consider a permanent increase in foreign aid flows to the above benchmark specifications. Specifically, the transfer from abroad is tied to the scale of the economy, and increases from 0% of GDP in the initial steady-state to 5% of GDP in the new steady-state (an increase in θ from 0 to 0.05). However, this aid may be tied to new investment in public capital ($\lambda = 1$), representing the case of a “productive” transfer, or it may be untied ($\lambda = 0$), representing the case of a “pure” transfer from abroad. The short-run and long-run responses of key variables in the recipient economy are reported in Rows 2 and 3 in Tables 1A - 1C, which correspond to the varying elasticity of substitution. The final column in the table summarizes the effects on economic welfare, measured by the optimized utility of the representative agent

$$W = \int_0^{\infty} \frac{1}{\gamma} C^{\gamma} e^{-\beta t} dt$$

where C is evaluated along the equilibrium path. These welfare changes are calculated as the percentage change in the initial stock of capital necessary to maintain the level of welfare unchanged following the particular shock. We will first discuss the long-run effects of the foreign aid shock (Tables 1-2) and then proceed to a discussion of the transitional dynamics generated by this shock (Figures 1-3).

3.1.1 *Tied Transfer*

The long run impact of a tied aid shock is reported in Rows 2 of Tables 1A-1C. Since the aid is tied to new investment in public capital, the implied long run increase in the stock of public capital increases the long run marginal product of private capital and generates a dynamic adjustment for its market price, q . However, the magnitude and direction of the initial response of q and its consequent

dynamic adjustment will depend crucially on the elasticity of substitution between the two types of capital stocks, σ .

Row 2 of Table 1B describes the standard case of the Cobb-Douglas production function. In the new steady state the ratio of public to private capital increases from 0.29 to 0.61, thereby generating a huge investment boom in infrastructure. The increase in the stock of public capital increases the marginal productivity of private capital, thereby leading to a positive, though lesser accumulation of private capital. Although the transfer stimulates consumption through the wealth effect, (like the pure transfer) the higher long-run productive capacity has a greater effect on output, leading to a decline in the long-run consumption-output ratio from 0.60 to 0.56. The higher productivity raises the long-run growth rate to 1.94 %, while long run welfare improves by 9.83%, as indicated in the last column of Row 3. The increased accumulation of both private and public capital lead to a higher demand for external borrowing as a means of financing new investment in private capital and the installation costs of public capital. This results in an increase in the steady state debt-output ratio from 0.45 to 0.77, raising the borrowing premium to over 2.8%. However, this higher debt relative to output is sustainable since it is caused by higher investment demand rather than higher consumption demand. The long run increase in the economy's productive capacity (as measured by the higher stocks of public and private capital, and output) ensures that the higher debt is sustainable. This view has also been expressed by Roubini and Wachtel (1998).

For benchmark I (Table 1A, Row 2), since the elasticity of substitution between the two types of capital stock is very low ($\sigma = 0.33$), there is a large increase in q in order to induce the agent to increase private investment to complement the boom in public investment. The ratio of public to private capital increases from 0.44 to about 0.70. The large increase in q and the consequent investment boom turns the current account surplus into a deficit with the debt-GDP ratio increasing from -1.24 to 0.28 and the interest rate from 2.4% (3.6% below the world rate) to 7.04% (1.04% above the world rate). Consequently, the consumption-output ratio goes down from 0.78 to 0.64, indicating a large substitution away from consumption. The steady-state growth rate almost doubles from -0.6% to about 1.2% . There is a large long run welfare gain of about 50%.

In benchmark III, the polar case of perfect substitutability between public and private capital (Table 1C, Row 2), the long-run change in q is zero, hence the long-run increase in the ratio of public to private capital from 0.27 to 0.58 can be attributed mainly to the boom in public capital brought about by the tied foreign aid shock. As a result, the long-run interest rate remains unchanged. Since there is no long-run effect on q , the boom in public investment does not crowd out consumption as before, but leads to a slight increase in the consumption-output ratio from 0.50 to 0.51. Consequently, the long-run debt position of the economy improves from 1.11 to 1.04. Another effect of q not changing in the long run is that the growth rate remains unchanged at 2.35%. The tied transfer also entails a long-run welfare loss of 2.43%. However, even though the tied aid in this case does not have long-run effects on certain key variables, it does generate a dynamic adjustment that will be discussed below in section 3.2.

3.1.2 *Pure Transfer*

A permanent pure transfer shock, i.e., an aid flow not tied to any investment activity, does not generate any transitional adjustment and nor does it have any long run effects on the key variables in the economy except consumption and welfare (Tables 1A-1C, Row 3). The pure transfer only raises consumption proportionately and also long run welfare. For example, for benchmark I, the consumption-output ratio goes up from 0.78 to 0.83, and from 0.60 to 0.65 for benchmark II. For benchmark III, it goes up from 0.51 to 0.56. However, even though long run welfare increases due to an untied aid flow, the gains increase with the elasticity of substitution. For benchmark I, the gain in welfare is 6.4%, while it is 8.3% and 9.8% for benchmarks II and III respectively.

3.2. *The Effectiveness of Foreign Aid and the Elasticity of Substitution*

The dependence of the efficacy of a tied aid program on the elasticity of substitution in production in the recipient country is an important question. Some indication of this is provided in the three panels of Table 1, and this is further considered in Table 2, where the range of the elasticity of substitution is expanded to cover the range $0.1 \leq \sigma \leq \infty$. The Cobb-Douglas production function

is indicated in bold, while the shaded area reflects values of σ that lie within plausible sampling errors of $\sigma = 1$. One of the highlights of this table is that the effects of the tied transfer on the equilibrium debt-output ratio, the equilibrium growth rate and welfare are highly sensitive to relatively minor changes in σ from this benchmark value. Thus, for example, if a researcher estimates $\sigma = 1$ with a standard error of 0.1 -- a tight estimate -- with 95% probability the implied welfare gain of 9.83% could be as high as 14.8% or as low as 7%

From Table 2 we find that as σ increases, a tied aid program leads to generally higher long run increases in the ratio of public to private capital and smaller increases in q . Due to the smaller increases in q as σ increases, the increase in private investment is also reduced. This leads to less borrowing, reflected by a decline in the increase in the equilibrium interest rate and debt-GDP ratio as σ increases. In fact, as σ approaches infinity, the current account actually improves. The crowding out of consumption also declines as σ increases, thereby reflecting lower induced private investment due to higher substitutability in production. An increase in σ also reduces the positive effect of the tied aid program on growth and welfare: the long run gains from both growth and welfare decline as substitutability in production goes up.

The above results lead us to believe that insofar as its effect on long run growth and welfare is concerned, a tied aid program is more effective in countries with a low elasticity of substitution in production. This observation complements the recent findings of Duffy and Papageorgiou (2000) that less developed or poor countries have elasticities of substitution that are significantly below unity and developed or richer countries have elasticities that are significantly above unity. In such a scenario, our analysis shows that a tied aid program may be more effective for poor countries than for their richer counterparts.

3.3. *Transitional Dynamics*

The transitional dynamic responses of the economy to a tied aid program are illustrated in Figures 1-3, corresponding to, low ($\sigma = 0.33$), Cobb-Douglas ($\sigma = 1$), and high factor substitutability ($\sigma \rightarrow \infty$) respectively. The basic phase diagram, showing the stable adjustment path in $k_g - n$ space are graphed in Figs 1.1, 2.1 and 3.1. For low degrees of substitutability, k_g and n

both increase approximately proportionately, reflecting the paths of the differential growth rates. The initial stimulus to public capital raises its initial growth rate to over 2.7%, after which it declines monotonically toward the new equilibrium of 1.2%. By contrast, private capital adjusts only gradually. Indeed, after increasing on impact to 1.04%, it declines marginally, before the stimulating effect of the higher public capital has its full impact and eventually raises its growth rate toward the equilibrium. With public capital growing uniformly faster than private capital, k_g is always increasing. The stimulus to investment, and the associated resource costs, raises borrowing and while this is mitigated by the amount of the transfer, national debt rises (or national credit falls) at a faster rate than domestic capital, so that borrowing costs rise as well. Over time, as the growth rate of private capital catches up, borrowing costs and the need to accumulate further debt are mitigated, and the transitional path tilts in favor of the relative accumulation of public capital.

As the elasticity of substitution increases, the curvature of the adjustment path increases. The higher the degree of substitution between the two types of capital, the more the transfer increases the initial growth rate of public capital relative to that of private capital.¹⁴ At the same time, the rate of debt accumulation increases raising borrowing costs. Over time, as the growth rate of public capital declines and that of private capital increases, foreign borrowing and borrowing costs fall. For a very high elasticity of substitution, we get very rapidly increasing debt and borrowing costs during the early phases of the transition. However, over time, these inhibit borrowing, which declines and in the limiting case where the two types of capital are perfect substitutes, n ultimately returns to its initial level.

The contrasting transitional paths of the four growth rates ϕ_K, ϕ_G, ϕ_Y and ϕ_C toward their common long-run growth rate are shown in Figs. 1.6, 2.6 and 3.6. In all cases, the stimulus to public capital raises its initial growth rate substantially, after which it declines monotonically. By contrast, private capital adjusts only gradually. The growth rate of output is an average of the growth rates of the two capital stocks. The fact that the growth rate of output initially doubles from 1.37% to 2.72%, in the case of the Cobb-Douglas production function, is of interest and is consistent with the experiences of some of the recipient countries in the European Union. Finally, the growth rate of

¹⁴ Care must be exercised in comparing the slopes of the $n - k_g$ loci in Figs. 1.1 - 1.3, as the units vary.

consumption is unaffected on impact and responds only gradually. The reason for this becomes evident by recalling the growth rate of consumption,

$$\frac{\dot{C}}{C} = \phi_c = \frac{r(n) - \beta}{1 - \gamma}$$

and the fact that it depends upon the sluggishly evolving debt-capital ratio, n .

An alternative perspective on the transitional adjustment paths can be obtained by looking at the transitional dynamics of q , the market price of private capital, depicted in figures 1.2, 2.2, and 3.2. When σ is low (fig. 1.2), the initial jump in q has to be large, in order to induce the required private investment to complement the long run increase in the stock of public capital. The large increase in q results in a large initial increase in private investment which leads to the initial decline in k_g . Thereafter q declines towards its new steady state level and the resultant decrease in the rate of increase of private investment causes k_g to gradually increase toward its new higher steady-state level. The same observations carry over to the case of $\sigma = 1$ (fig. 2.2). However, in this case the required initial jump in q is smaller. In the case of perfect substitutability (fig. 3.2), q decreases instantaneously to accommodate the implied boom in public investment. The consequent fall in private investment leads to the initial monotonic increase in k_g and n . However, the higher stock of public capital raises the marginal product of private capital, and eventually both q and private investment start rising. Due to perfect substitutability, the long run increase in q must be zero; hence q eventually returns to its initial steady state level, and so does n ; after the initial increase, it gradually declines back to its original equilibrium level.

The time paths for the consumption-capital ratio and consumption output ratio are depicted in figures 1.3-1.4, 2.3-2.4, and 3.3-3.4 respectively. For low values of σ ($= 0.33$ and 1), there is an initial upward jump in consumption due to the wealth effect created by the initial upward jump in q . Thereafter, as private capital accumulation and output increases, the consumption-capital and consumption-output ratios decline monotonically toward their respective long run equilibrium values. However, when there is perfect substitutability of the two types of capital, the initial downward jump in q creates a negative wealth effect and the consumption-capital and consumption-output ratios jump down slightly on the incidence of the shock. However, as q gradually increases,

the wealth effect becomes positive and consumption increases in transition to its new higher equilibrium level.

The dynamics for the debt-GDP ratio are depicted in figures 1.5, 2.5, and 3.5. For low values of σ ($= 0.33$ and 1), the implied capital accumulation increases the debt-GDP ratio monotonically towards its new higher steady-state level. However, in the case of perfect substitutability, the increase in the debt-GDP ratio is reversed due to the decrease in private capital accumulation and borrowing, and the debt position of the economy improves as the debt-GDP ratio now monotonically decline to its new lower steady-state level.

4. Co-financing

Several aid programs call for co-financing by the domestic government. In Table 3 we compare the welfare effects of the tied and pure transfers with two alternative forms of cofinancing. In the first, the government receives a tied transfer of 2.5% of its income, which it must match with an equal increase in its expenditure; in the second it must match an untied transfer. In all four cases, the economy is experiencing a 5% increase in expenditure.

For low or medium elasticity of substitution the tied transfer (TT) is superior to the pure transfer (PT), where as for a high σ this ordering is reversed, as we have seen. In all cases the matched tied transfer (MTT) is dominated by TT. This is because the MTT involves making the size of the government sector too large. While the matched pure transfer (MPT) is never dominant, it is superior to PT in the case where $\sigma = 0.33$ and it is superior to TT as $\sigma \rightarrow \infty$.¹⁵

5. Welfare Sensitivity to Investment Costs and Capital Market Imperfections

While the above parameters represent a plausible description of a small poorly endowed open economy, some of the welfare implications are dependent upon this characterization. Table 4 conducts some sensitivity analysis. Specifically, we compare the welfare gains from a tied and an untied aid program In response to variations in the following three factors:

¹⁵ Chatterjee, Sakoulis, and Turnovsky (2001) address the question of optimal co-financing in the case of the Cobb-Douglas production. The analogous exercise can be pursued here.

- (i) the cost of installing public capital (h_2);
- (ii) the cost of borrowing from international capital markets (a);
- (iii) the elasticity of substitution between public and private capital (σ).

Specifically, each table addresses the following question: For a given cost of installing public capital, what are the gains from a tied and untied aid program when (i) the cost of borrowing increases (measured by an increase in a across a row) and (ii) the elasticity of substitution increases (measured by an increase in σ down a column). Therefore, $a = 0.02$ implies a low cost of borrowing from international capital markets, and $a = 10$ implies that the agent has virtually little or no access to international capital markets. The range of σ we consider is from 0.33 to 4. We consider three values for investment costs for public capital, with $h_2 = 1, 15$, and 50 signifying low, medium, and very high costs of installing public capital. For example, in Table 4A, when $h_2 = 1$, $a = 0.02$, and $\sigma = 0.8$, the welfare gain from an untied transfer is 8.27%, while from a tied transfer it is 34.52%. The following observations can be drawn from panels 4A-4C.

(i) An increase in the elasticity of substitution always increases the welfare gains resulting from a *pure* transfer. It reduces the welfare gains resulting from a *tied* transfer, and indeed it may lead to a welfare loss, if the installation costs associated with public capital are sufficiently large. The effects of tied transfers are much more sensitive to σ than are those of pure transfers, and the sensitivity of both decreases with a .

The intuition underlying this key result is as follows. A pure transfer has no effect on the stocks of public or private capital; all that happens is that consumption increases, raising the C/Y ratio. The higher elasticity of substitution raises the level of output attainable from given stocks of capital, thereby raising consumption and welfare uniformly. If the transfer is tied, the transfer increases the rate of investment in public capital. With a low elasticity of substitution this requires an approximately corresponding increase in private capital, leading to a large increase in output, consumption, and benefits. As the elasticity of substitution increases, the higher public capital is associated with a larger decline of private capital, so that the increase in output, consumption, and

welfare declines. With very high installation costs, the tied transfer is committing the recipient economy to devote a large portion of its resources to the costly task of installation, thereby making it worse off.

Other observations can be seen from Table 4:

(ii) Except if σ is very low, the benefits from pure aid decrease with the cost of borrowing. As long as tied aid yields positive benefits, these decrease with the cost of borrowing. In the cases where σ, h_2 are both high, so that tied aid leads to welfare losses, these losses decline with the cost of borrowing.

(iii) Irrespective of the cost of borrowing, and elasticity of substitution, the benefits from tied aid decrease and those from untied aid increase with installation costs.

(iv) Even though the magnitude of welfare gains from tied aid are generally higher than those from untied aid, for high values of $\sigma (=4)$, an untied aid program is strictly better than a tied aid program. This observation, along with the one made in (i) above, suggest that countries with a low elasticity of substitution between public and private capital in production may be better off with tied aid, whereas countries having a high degree of substitutability may benefit more from aid programs that are untied.

(v) When installation costs are high ($h_2 = 50$), an untied transfer is better than a tied transfer even for $\sigma = 1$ (the Cobb-Douglas case), when the latter is unambiguously welfare-deteriorating.

(vi) If we consider $\sigma = 1$ and $a = 0.10$ as benchmark values, then even small deviations of σ from the benchmark (in the range 0.8-1.2), lead to substantial variations in welfare changes from both types of aid programs, irrespective of the cost of adjustment.

6. Conclusions

This paper has characterized the effectiveness of a tied and untied aid program and the dynamic response it evokes in the recipient economy. We find that the long run impact of a tied aid

program and the direction of transitional dynamics it generates depend crucially upon the elasticity of substitution in production. Our numerical simulations suggest that tied aid is more effective in economies with a low degree of substitution between factors of production. Moreover, the welfare gains from a tied or untied aid shock are sensitive to the substitutability of inputs, capital market imperfections, and costs of adjustment. These findings imply that when donors decide on whether a particular aid program should be tied to an investment activity, careful attention must be paid to the recipient's opportunities for substitution in production, its access to world capital markets, and the costs of installing the particular type of capital to which the aid will be tied.

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TABLE 1: Responses to a Permanent Transfer Shock**1A. Benchmark I: Low Substitutability in Production ($\sigma = 0.33$)**

	\tilde{k}_g	\tilde{r} %	\tilde{C}/Y	\tilde{N}/Y	$\phi_K(0)$ %	$\phi_G(0)$ %	$\phi_Y(0)$ %	$\phi_C(0)$ %	$\tilde{\phi}$ %	$\Delta(W)$ %
Benchmark I $\theta = 0, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.437	2.42	0.778	-1.24	-0.60	-0.60	-0.60	-0.60	-0.60	---
Tied transfer $\theta = 0.05, \lambda = 1,$ $\bar{g} = 0.05, \tau = 0.15$	0.696	7.04	0.643	0.29	1.04	2.73	1.99	-0.60	1.22	+50.0
Pure-transfer $\theta = 0.05, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.437	2.42	0.828	-1.24	-0.6	-0.60	-0.60	-0.60	-0.60	+6.42

1B. Benchmark II: Unitary Substitutability in Production ($\sigma = 1$)

	\tilde{k}_g	\tilde{r} %	\tilde{C}/Y	\tilde{N}/Y	$\phi_K(0)$ %	$\phi_G(0)$ %	$\phi_Y(0)$ %	$\phi_C(0)$ %	$\tilde{\phi}$ %	$\Delta(W)$ %
Benchmark II $\theta = 0, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.291	7.42	0.60	0.45	1.37	1.37	1.37	1.37	1.37	---
Tied transfer $\theta = 0.05, \lambda = 1,$ $\bar{g} = 0.05, \tau = 0.15$	0.61	8.84	0.561	0.774	1.70	6.74	2.71	1.37	1.938	+9.83
Pure-transfer $\theta = 0.05, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.291	7.42	0.651	0.45	1.37	1.37	1.37	1.37	1.37	+8.32

1C. Benchmark III: Perfect Substitutability in Production ($\sigma \equiv \infty$)

	\tilde{k}_g	\tilde{r} %	\tilde{C}/Y	\tilde{N}/Y	$\phi_K(0)$ %	$\phi_G(0)$ %	$\phi_Y(0)$ %	$\phi_C(0)$ %	$\tilde{\phi}$ %	$\Delta(W)$ %
Benchmark III $\theta = 0, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.269	9.87	0.509	1.11	2.35	2.35	2.35	2.35	2.35	---
Tied transfer $\theta = 0.05, \lambda = 1,$ $\bar{g} = 0.05, \tau = 0.15$	0.577	9.87	0.513	1.04	2.08	8.69	2.50	2.35	2.35	-2.43
Pure-transfer $\theta = 0.05, \lambda = 0,$ $\bar{g} = 0.05, \tau = 0.15$	0.269	9.87	0.559	1.11	2.35	2.35	2.35	2.35	2.35	+9.82

TABLE 2

Sensitivity of Permanent Responses to the Elasticity of Substitution

	$d\tilde{k}_g$	$d\tilde{q}$	$d\tilde{r}$ % pts	$d(\tilde{C}/Y)$	$d(\tilde{N}/Y)$	$d\tilde{\phi}$	$\Delta(W)$ %
$\sigma = 0.1$	0.113	0.32	5.26	-0.122	1.599	2.10	70.93
$\sigma = 0.5$	0.300	0.21	3.49	-0.108	1.071	1.39	32.10
$\sigma = 0.8$	0.318	0.12	1.96	-0.059	0.501	0.78	14.81
$\sigma = 1$	0.319	0.09	1.42	-0.040	0.322	0.57	9.83
$\sigma = 1.2$	0.319	0.07	1.09	-0.029	0.218	0.44	6.96
$\sigma = 1.5$	0.318	0.05	0.79	-0.019	0.130	0.32	4.45
$\sigma = 4$	0.312	0.01	0.22	-0.002	-0.023	0.09	-0.36
$\sigma \cong \infty$	0.308	0.00	0.00	0.004	-0.075	0.00	-2.43

TABLE 3: Co-financing Tradeoffs**3A. Low Substitution in Production: $\sigma = 0.33$**

	$\% \Delta(W)$
Tied transfer (TT) $\theta = 0.05, \lambda = 1, \bar{g} = 0.05$	50.05
Pure transfer (PT) $\theta = 0.05, \lambda = 0, \bar{g} = 0.05$	6.42
Matched tied transfer (MTT) $\theta = 0.025, \lambda = 1, \bar{g} = 0.075$	44.90
Matched pure transfer (MPT) $\theta = 0.025, \lambda = 0, \bar{g} = 0.075$	26.82

$$\text{TT} > \text{MTT} > \text{MPT} > \text{PT}$$

3B. Unitary Substitution in Production: $\sigma = 1$

	$\% \Delta(W)$
Tied transfer (TT) $\theta = 0.05, \lambda = 1, \bar{g} = 0.05$	9.83
Pure transfer (PT) $\theta = 0.05, \lambda = 0, \bar{g} = 0.05$	8.32
Matched Tied transfer (MTT) $\theta = 0.025, \lambda = 1, \bar{g} = 0.075$	5.07
Matched Pure transfer (MPT) $\theta = 0.025, \lambda = 0, \bar{g} = 0.075$	5.35

$$\text{TT} > \text{PT} > \text{MPT} > \text{MTT}$$

3C. Perfect Substitution in Production: $\sigma \cong \infty$

	$\% \Delta(W) \%$
Tied transfer (TT) $\theta = 0.05, \lambda = 1, \bar{g} = 0.05$	-2.42
Pure transfer (PT) $\theta = 0.05, \lambda = 0, \bar{g} = 0.05$	9.82
Matched Tied transfer (MTT) $\theta = 0.025, \lambda = 1, \bar{g} = 0.075$	-7.34
Matched Pure transfer (MPT) $\theta = 0.025, \lambda = 0, \bar{g} = 0.075$	-1.27

$$\text{PT} > \text{MPT} > \text{TT} > \text{MTT}$$

TABLE 4
Welfare Sensitivity to Installation Costs, Capital Market Imperfections, and Degree of
Substitutability
($\theta = 0$ to $\theta = 0.05$; $\lambda = 1$)

4A. Low Installation Costs ($h_2 = 1$)

	$a = 0.02$		$a = 0.10$		$a = 10$	
	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$
$\sigma = 0.33$	5.31%	90.39%	6.33%	56.55%	6.64%	48.47%
$\sigma = 0.8$	8.27%	34.52%	7.70%	21.28%	7.57%	19.84%
$\sigma = 1$	9.79%	26.54%	8.06%	16.27%	7.73%	15.32%
$\sigma = 1.2$	11.16%	22.12%	8.32%	13.38%	7.82%	12.64%
$\sigma = 4$	19.33%	12.19%	9.13%	6.21%	8.08%	5.75%

4B. Medium Installation Costs ($h_2 = 15$)

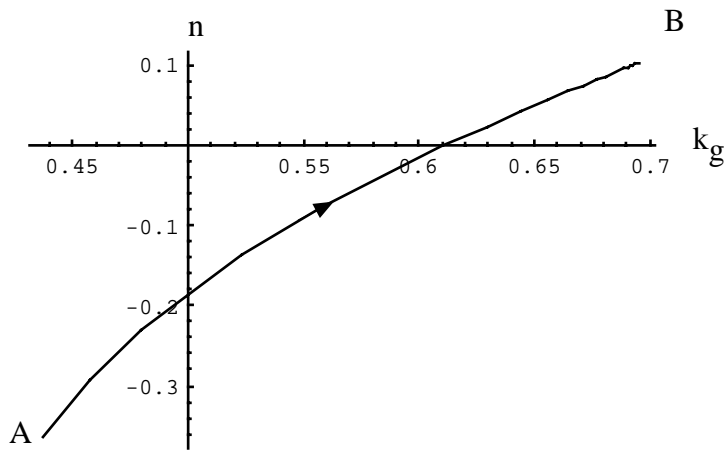
	$a = 0.02$		$a = 0.10$		$a = 10$	
	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$
$\sigma = 0.33$	5.38%	84.76%	6.42%	50.05%	6.75%	42.06%
$\sigma = 0.8$	8.52%	26.90%	7.91%	14.81%	7.77%	13.86%
$\sigma = 1$	10.17%	17.99%	8.32%	9.83%	7.96%	9.47%
$\sigma = 1.2$	11.66%	12.72%	8.59%	6.96%	8.07%	6.87%
$\sigma = 4$	21.09%	-2.67%	9.50%	-0.36%	8.37%	0.07%

4C. High Installation Costs ($h_2 = 50$)

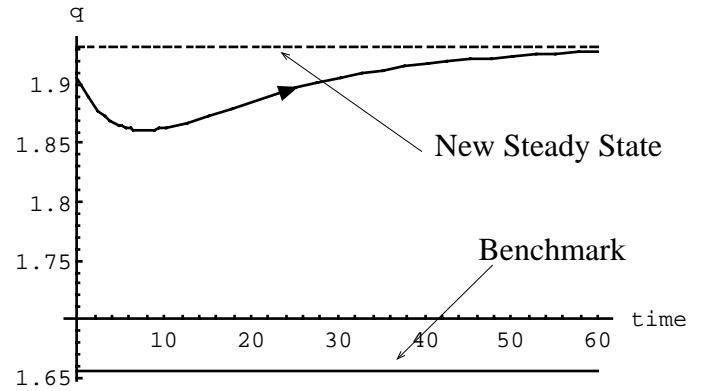
	$a = 0.02$		$a = 0.10$		$a = 10$	
	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$	$\lambda = 0$	$\lambda = 1$
$\sigma = 0.33$	5.56%	69.87%	6.68%	32.95%	7.03%	25.33%
$\sigma = 0.8$	9.22%	5.51%	8.51%	-2.61%	8.35%	-1.99%
$\sigma = 1$	11.24%	-6.55%	9.02%	-7.56%	8.60%	-6.08%
$\sigma = 1.2$	13.16%	-14.82%	9.38%	-10.46%	8.76%	-8.53%
$\sigma = 4$	27.30%	-52.59%	10.59%	-16.65%	9.20%	-15.15%

Dynamic Response to Permanent Tied Aid Shock

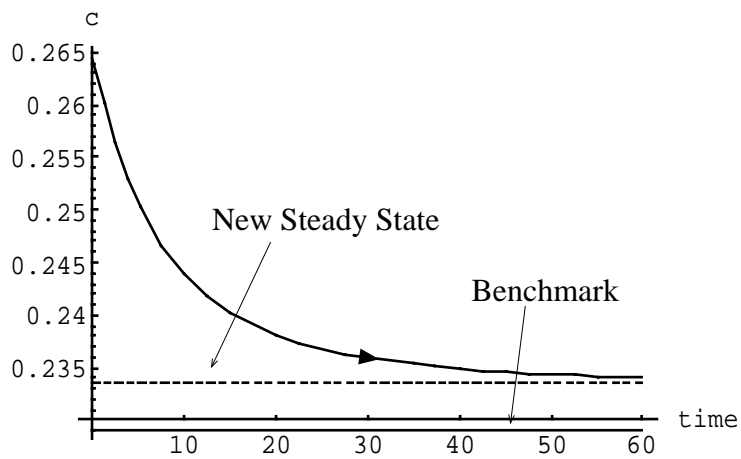
Figure 1: Low Substitutability of Inputs: $\sigma=0.33$



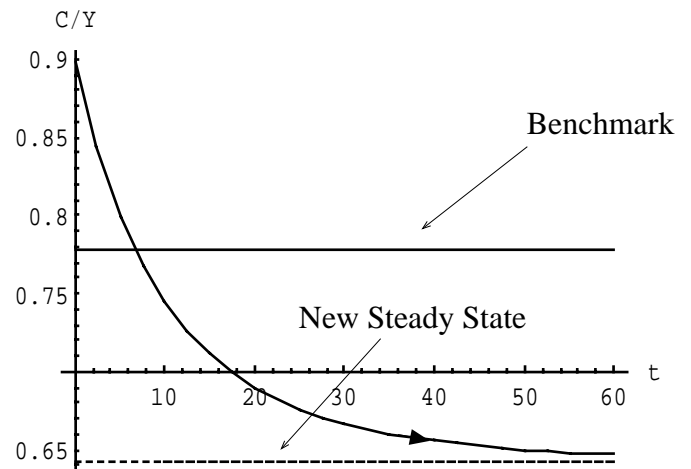
1.1 Phase Diagram



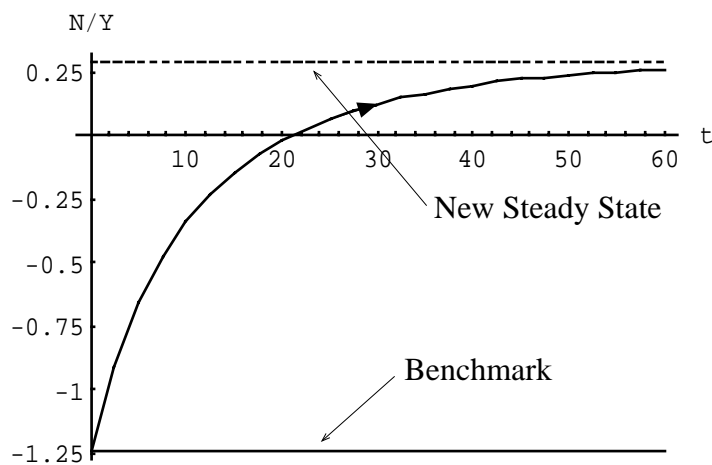
1.2 Market price of Private Capital



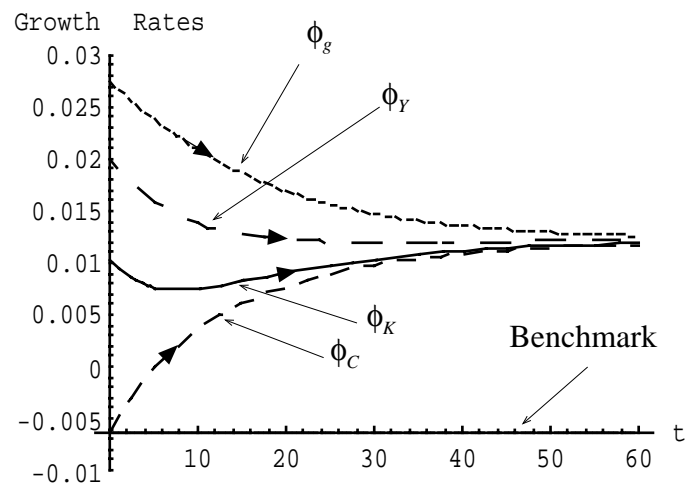
1.3 Consumption-Private Capital Ratio



1.4 Consumption-Output Ratio



1.5 Debt-Output Ratio



1.6 Growth Rates

Figure 2: Unitary Elasticity of Substitution: $\sigma=1$

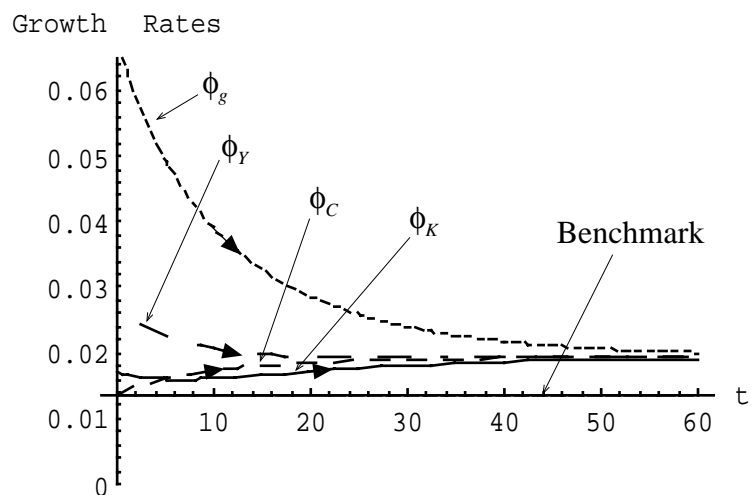
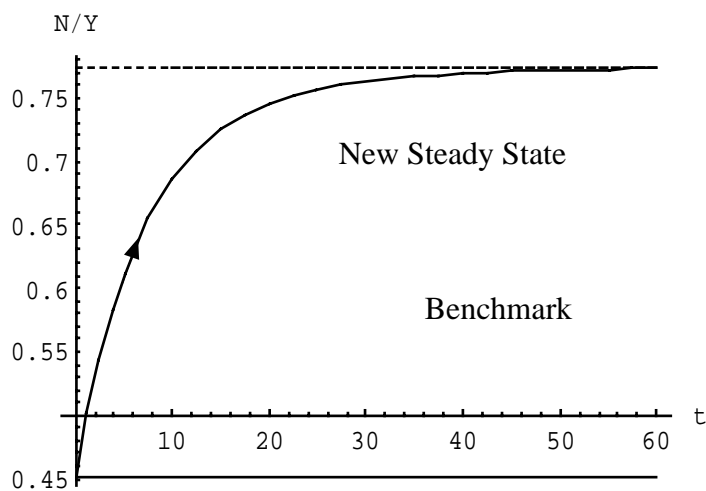
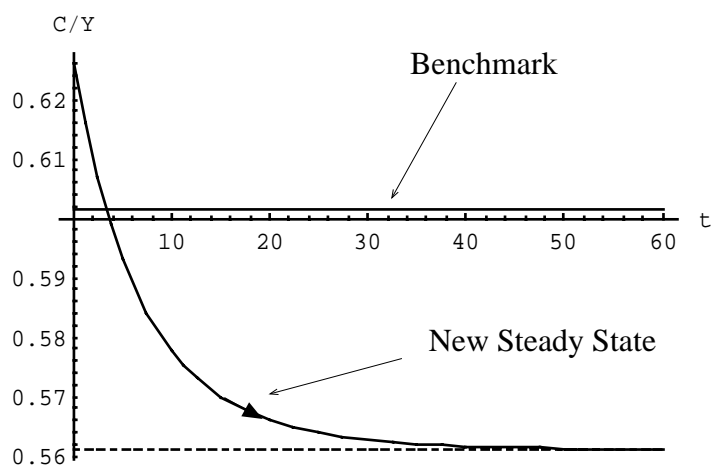
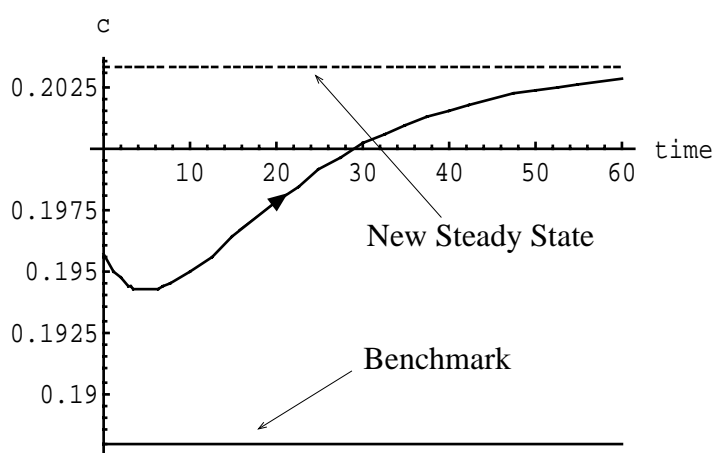
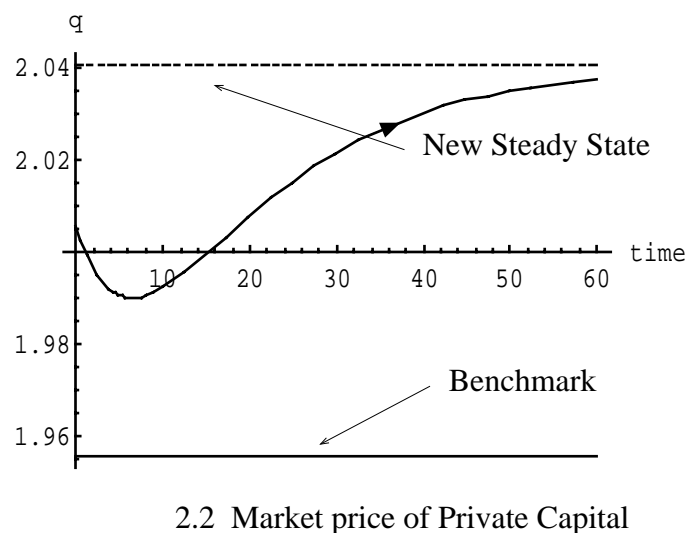
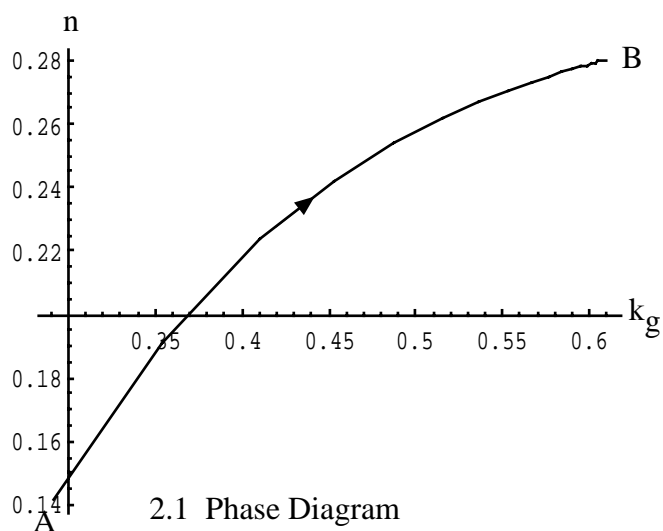


Figure 3: Perfect Substitutability of Inputs: $\sigma \rightarrow \infty$

